Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 18, 2002*
Accepted: *January 7, 2003*

Possible explanation of the discrepancy of the light-cone QCD sum rule calculation of $g_{D^*D\pi}$ coupling with experiment

Damir Becirevic
Dip. di Fisica, Università di Roma "La Sapienza"
Piazzale Aldo Moro 2, I-00185 Rome, Italy
E-mail: Damir.Becirevic@roma1.infn.it

Jerome Charles
Centre de Physique Théorique, CNRS Luminy, Case 907[*]
F-13288 Marseille Cedex 9, France
E-mail: charles@cpt.univ-mrs.fr

Alain Le Yaouanc, Luis Oliver, Olivier Pène and J.C. Raynal
Université de Paris Sud, Centre d'Orsay, LPT (Bât.210)[†]
F-91405 Orsay Cedex, France
E-mail: leyaouan@qcd.th.u-psud.fr, luis.oliver@th.u-psud.fr, olivier.pene@th.u-psud.fr, ???

Abstract: The introduction of an explicit *negative* radial excitation contribution in the hadronic side of the light cone QCD sum rule (LCSR) of Belyaev, Braun, Khodjamirian and Rückl, can explain the large experimental value of $g_{D^*D\pi}$, recently measured by CLEO. At the same time, it considerably improves the stability of the sum rule when varying the Borel parameter M^2.

Keywords: Heavy Quarks Physics, Sum Rules, Phenomenological Models.

[*] Unite propre de Recherche du CNRS - UPR 7061.
[†] Unité mixte de Recherche du CNRS - UMR 8627.

JHEP01(2003)009

Contents

1. **Recalling the difficulty** 1

2. **An appealing solution: a large negative radial excitation contribution to the LCSR** 2

3. **Check of $D \to \pi \ell \nu$ semileptonic decay** 5

4. **Check of annihilation constants and Adler-Weisberger sum rule** 6

5. **Conclusion** 7

1. Recalling the difficulty

The discrepancy of the LCSR prediction for $g_{D^*D\pi}$ with the recent experimental result by CLEO is a striking and puzzling one. We recall the reader that the LCSR prediction made in ref. [1] reads $g_{D^*D\pi} = 12.5$, which got even smaller after the radiative corrections have been included [2], namely $g_{D^*D\pi} = 10.5 \pm 3.0$. On the other hand the experimental value is $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$ [3]. Thus the discrepancy is large.[1] The above LCSR result has been very carefully discussed, the calculation has been improved several times, and it has also been verified by using other sum rule techniques [4]. Although the QCD sum rule approach certainly suffers from large uncertainties, in most cases a good agreement with experiment has been obtained. Therefore, we cannot be satisfied with concluding by a comfortable attitude of skepticism against the whole sum rule approach. Still more puzzling is the fact that this quantity does not seem a priori to have anything particularly exotic. In addition, the other theoretical approaches do not show such a discrepancy for this quantity. A careful discussion meant to reduce the uncertainties presented by quark models, and performed in the framework of Dirac equation [5], prior to the experimental measurement, has led to a result $g_{D^*D\pi} \simeq 18$. It should be stressed that this result has been obtained in the heavy quark limit (for the latest quark model discussion of $g_{D^*D\pi}$ see ref. [6]). The recent (quenched) lattice QCD calculation has led to $g_{D^*D\pi} = 18.8 \pm 2.3^{+1.1}_{-2.0}$ [7]. It is therefore important to understand the specific difficulty which the standard sum rule approach seems to encounter in this case.[2] In ref. [9] it has been noted that the simple quark-hadron duality ansatz which works in the one-variable dispersion relations might be too crude for the double dispersion relation.

[1]Notice that we speak here of the discrepancy of central values. Of course, there are errors in the sum rule estimate, but it is difficult to draw strict conclusions from them, since their estimate is itself, of course, uncertain.

[2]A tentative proposal to solve the difficulty of the LCSR prediction of $g_{D^*D\pi}$ has been discussed in [8].

2. An appealing solution: a large negative radial excitation contribution to the LCSR

In sum rule calculations it is usual to assume that the higher state contributions can be included in the perturbative estimate of a continuum.[3] In other words, one does not include an *explicit* contribution of an isolated excited state. The first reason is probably pragmatical: it seems unnecessary to recourse to large excitation contributions if the stability criterion can be well satisfied without them. The other, more theoretical, reason is that the excitation contributions are exponentially suppressed with respect to the lowest state by the Borel procedure, so that one can sufficiently account for them by the rough procedure of the perturbative treatment of the continuum. Indeed, in the LCSR approach of ref. [1], this point of view has been adopted. We suggest, instead, that this neglect of an explicit radial excitation contribution may be the origin of the above discrepancy between the LCSR prediction and the experimental value for $g_{D^*D\pi}$. Of course, we are aware that, as explained in Belyaev et al. [1], in their method such contributions are suppressed by the Borel exponential. Our claim is that, in this particular case, the Borel suppression is not sufficient to allow to neglect them. The inclusion of the explicit radial excitation contribution to the hadronic side of the LCSR (often referred to as the left hand side — l.h.s. — of the sum rule) also offers an appealing explanation for the failure of the LCSR prediction.

A first indication in favor of this new proposal is that, after including a radial excitation, the stability of the sum rule, under the variation of the Borel parameter M^2, is improved. This criterion is however subject to an uncertainty since we do not really know what is the accuracy of the calculation of the theoretical r.h.s. It would be therefore good to present cross-checks of our assumption, which is what we will do in the next sections.

Before embarking on those issues, let us first explain why the assumption of a large radial excitation coupling to the sum rule, *with a negative sign* relative to the ground state contribution, seems so interesting. Since the purpose of this letter is to communicate our proposal, and not to make the best numerics, we leave apart for the moment the radiative corrections and write the sum rule of Belyaev et al. as:

$$g_{D^*D\pi} = \frac{f(M^2)}{f_D f_{D^*}}\,, \tag{2.1}$$

where the function $f(M^2)$ is the r.h.s. of eq. (44) of ref. [1], namely

$$f(M^2) = \frac{m_c^2}{m_D^2 m_{D^*}}\; f_\pi \phi_\pi \left(\frac{1}{2}\right)\; M^2 \exp\left(\frac{m_D^2 + m_{D^*}^2}{2M^2}\right) \left[e^{-m_c^2/M^2} - e^{-s_0/M^2}\right] + \cdots \tag{2.2}$$

We do not write the higher twist terms since it would make the expression lengthy and would not help understanding our proposal. Those terms are numerically very important and are indeed included in our calculation. It is then found that, within the Borel window $2\ \mathrm{GeV}^2 < M^2 < 4\ \mathrm{GeV}^2$ determined by the standard criteria, the function $f(M^2)$ is

[3] Except for the critical case of quark masses in the pseudoscalar correlator method, or in general for some refined calculations like the ones discussed in ref. [10].

monotonously decreasing, and the variation is as large as 20%. Therefore, there is no truly good plateau. The authors quote an average

$$\overline{f}(M^2)\big|_{2\ \text{GeV}^2<M^2<4\ \text{GeV}^2} = 0.51 \pm 0.05\ \text{GeV}^2\,, \tag{2.3}$$

which then yields the central value :

$$g_{D^*D\pi} = 12.5\,, \tag{2.4}$$

indeed much too low. Let us now introduce a radial excitation contribution to the hadronic l.h.s. of eq. (44) of ref. [1], or equivalently write:

$$g_{D^*D\pi} = \frac{1}{f_D f_{D^*}}\left[f(M^2) - R_{D'}\exp\left(-\frac{m_{D'}^2 - m_D^2}{2M^2}\right) - R_{D^{*\prime}}\exp\left(-\frac{m_{D^{*\prime}}^2 - m_{D^*}^2}{2M^2}\right)\right]. \tag{2.5}$$

Note that we have two extra contributions: either the D or the D^* is excited; they are denoted as (D', D^*) and $(D, D^{*\prime})$, leading respectively to :

$$R_{D'} = \left(\frac{m_{D'}}{m_D}\right)^2 f_{D'} f_{D^*} g_{D^*D'\pi}\,, \qquad R_{D^{*\prime}} = \frac{m_{D^{*\prime}}}{m_{D^*}} f_D f_{D^{*\prime}} g_{D^{*\prime}D\pi}\,. \tag{2.6}$$

We assume that the higher $(D', D^{*\prime})$, (D'', D^*), $\ldots$ contributions are still included in the continuum part of the model. The completion of the procedure requires also a new value of s_0, since the lowest radial contributions are no more comprised in the continuum part, but are handled separately.

For simplicity, since there is no sense in requiring too much precision, we will assume:

$$m_{D'} = m_{D^{*\prime}}\,, \qquad f_{D'} = f_{D^{*\prime}}\,, \tag{2.7}$$

the spin-spin effect being expected to decrease for higher states. We also assume that $g_{D^*D'\pi} = g_{D^{*\prime}D\pi} = g'$, which is more questionable, since the ground state is present, and D differ appreciably from D^*. This, however, should not alter the qualitative conclusion that we obtain. Note also that the difference between the 0^- and 1^- states would disappear in the heavy quark limit. With these assumptions, we have

$$\frac{R_{D'}}{R_{D^{*\prime}}} = \frac{m_{D^{*\prime}} m_{D^*}}{m_D^2}\frac{f_{D^*}}{f_D}\,. \tag{2.8}$$

This equation (2.8) relates the two radial contributions since $R_{D'}/R_{D^{*\prime}}$ is directly calculable if we know the mass $m_{D^{*\prime}}$. The exponentials accompanying $R_{D'}/R_{D^{*\prime}}$ are the trace of the Borel suppression of radial excitations; they are *increasing* functions of M^2. Therefore, if we choose a *negative* $R_{D',D^{*\prime}}$, we can compensate for the decrease of $f(M^2)$ and thus simultaneously: (i) *improve the stability of the sum rule* and, (ii) *increase the magnitude of* $g_{D^*D\pi}$ (see figure 1 for illustration). We have of course to take a rather large radial contribution in order to get a significant effect. To avoid adding more freedom, we fix the excitation spectrum by a model calculation which is completely independent. Using Dirac equation, with numbers taken from our previous treatment [5], we find in the heavy quark

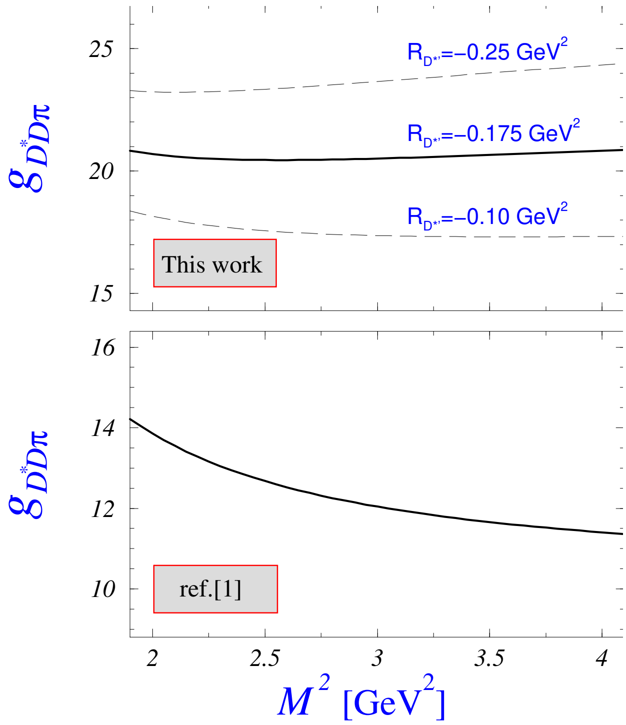

Figure 1: The effect of inclusion of the radially excited $D^{(*)\prime}$-meson contributions to the hadronic part of the LCSR: In the lower figure the result of ref. [1] is reproduced (no explicit radial excitations considered); In the upper curve we include the radial excitations as indicated in eq. eq. (2.5) with the value of $R_{D^{*\prime}}$ varied in the range specified in eq. (2.9). The latter range arises from the requirement of the stability of the sum rule eq. (2.5) to be within the 5% level, when the Borel parameter is varied as $2\ \mathrm{GeV}^2 \leq M^2 \leq 4\ \mathrm{GeV}^2$.

limit that the excitation energy is 0.5 GeV, and therefore the mass of the excitation is 2.5 GeV. Let us recall that the large width [11] expected for such a state makes it difficult to observe.

The remarkable fact is the following: taking a negative $R_{D^{*\prime}}$ ($R_{D'}$), and imposing that the variation of $g_{D^*D\pi}$ is 5% or less in the allowed window for M^2, (instead of the previous 20%, in the same Borel window), we find that $R_{D^{*\prime}}$ must be in the range

$$-0.25\ \mathrm{GeV}^2 < R_{D^{*\prime}} < -0.1\ \mathrm{GeV}^2 \,. \tag{2.9}$$

Then, depending on the value of $R_{D^{*\prime}}$ in the above range eq. (2.9) the predicted value of $g_{D^*D\pi}$ varies as

$$17 \ < \ g_{D^*D\pi} \ < \ 25 \,. \tag{2.10}$$

It increases with $|R_{D^{*\prime}}|$. It is much larger than before. We obtain at the same time a *much better stability in* M^2, and a much larger coupling constant, *just in the desired range to agree with experiment.*

For $R_{D^{*\prime}} = -0.15\ \mathrm{GeV}^2$, we would obtain a perfect stability in M^2 (accuracy of the order of 1%) and the value $g_{D^*D\pi} = 19.4$, that is the value found in lattice QCD. But at the present stage, there is no sense in trying to get such an accuracy in stability, given the very rough precision of the calculation.

For the present note, we have adopted the parameters of ref. [1] for the allowed range of variation M^2 ; we choose also to keep the continuum threshold to be the same as s_0, the continuum threshold of the two-points annihilation sum rules proposed by these authors, although this may be questioned since we now take apart certain contributions of excited states; we do not indulge in any fine tuning. We have also not allowed the annihilation constants f_D, f_{D^*} to vary with M^2, but instead, we have taken the final value given in the same reference [1]: indeed, we should check whether f_D ,f_{D^*} as well as $g_{D^* D\pi}$ are independent of M^2. As we stated above, the inclusion of the radial excitation stabilizes the LCSR. This does not make the same effect in the sum rules for f_D, f_{D^*}. As we shall see, the radial excitation contribution is neither stabilizing the sum rule (w.r.t. the variation of the Borel parameter M^2), nor it creates a difficulty: it simply gives a negligible contribution, and does not modify the previously estimated values for f_D, f_{D^*}.

We note that the minus sign for the radial excitation contribution is crucial, and we would like therefore to have some argument for this from models. There is no very trustable model for radial excitation strong decay at the quantitative level. Nevertheless, it is indicative for our purpose that the rather standard model for strong interaction couplings, the non relativistic quark pair creation (QPC) model, used for calculational simplicity with harmonic oscillator wave functions, give precisely a stable *negative sign* for the product $f_D f_{D^{*\prime}} g_{D^{*\prime} D\pi}$ relative to the similar product for the ground state $f_D f_{D^*} g_{D^* D\pi}$. Hence, since, in the sum rule, the latter product is positive, this means that $R_{D^{*\prime}}$, which is precisely $f_D f_{D^{*\prime}} g_{D^{*\prime} D\pi}$ up to positive mass ratios, should be negative, as we have found in the sum rule.

3. Check of $D \to \pi\ell\nu$ semileptonic decay

We note that exactly the same quantity $R_{D^{*\prime}}$ will appear in a t-channel analysis of $D \to \pi\ell\nu$ decay. Indeed, using an unsubtracted dispersion relation, as required for the form factor [12], the addition of the residue of the pole corresponding to the radial excitation results in :

$$F_+(q^2) = \frac{1}{2m_{D^*}} \frac{f_{D^*} g_{D^* D\pi}}{1 - q^2/m^2_{D^*}} + \frac{1}{2 f_D m^2_{D^{*\prime}}} \frac{m_{D^*} R_{D^{*\prime}}}{1 - q^2/m^2_{D^{*\prime}}} + \cdots . \tag{3.1}$$

So the question is whether our "large" contribution of radial states in the LCSR leads to effects in the $D \to \pi\ell\nu$ which are compatible with the facts. The answer is first that its impact is rather small, and anyway rather in the right direction. Indeed, we find that for $R_{D^{*\prime}} = -0.15 \ \mathrm{GeV}^2$, allowing $g_{D^* D\pi} \simeq 19$, the residue of the radial excitation is almost one order of magnitude smaller than the ground state one :

$$F_+(q^2) = \frac{1.15}{1 - 0.25q^2} - \frac{0.14}{1 - 0.16q^2} + \cdots . \tag{3.2}$$

It should be noted that the ground state contribution alone would lead to a *too large rate* (the numerator should be about 0.8 at most instead of 1.15 to get the correct rate with only the first pole). Therefore, the negative radial contribution is not worrisome but, quite the contrary, it is not sufficiently large to balance the excess of the lowest state contribution. Other radial excitations, as well as the continuum, must contribute, which is in agreement with the conclusions of ref. [12]. Analogously, a negative contribution appears in the Becirevic-Kaidalov model [13], from remote singularities.

4. Check of annihilation constants and Adler-Weisberger sum rule

Another worry could be that this large contribution could contradict other sum rules concerning either the annihilation constants or the strong couplings. The annihilation constants f_D and f_{D^*} are determined from the QCD sum rules by using the P-P and V-V correlation functions, respectively. At the same time, an Adler-Weisberger sum rule constrains the strong interaction coupling constants. Since the latter is stronger, as we shall see, we begin with it and write it, in the case of $\pi + D \to \pi + D$ scattering, as :

$$\sum_{n=0}^{\infty} \frac{f_\pi^2}{4m_{D^{*(n)}}^2} |g_{D^{*(n)}D\pi}|^2 + \text{orbitally excited states} = 1\,, \tag{4.1}$$

where we have made explicit only the D^* ground state contribution and the one of its *radial* excitations ; the allowed orbital excitations are the 0^+ and 2^+. In ref. [5], we have estimated these contributions of the ground state and lowest orbitally excited states (D^{**}) to be 70%, whence

$$|g_{D^{*\prime}D\pi}| \frac{f_\pi}{2m_{D^{*\prime}}} < \sqrt{0.3} = 0.55\,. \tag{4.2}$$

Now, since we have $|R_{D^{(*)\prime}}| > 0.1$ GeV^2, we can deduce a lower bound on $f_{D^{*\prime}} = f_{D'}$ from eq. (2.6)

$$|f_{D'}| \gtrsim 0.02 \text{ GeV}\,. \tag{4.3}$$

This is one magnitude smaller than the ground state annihilation constant. Studying the QCD sum rules for f_D, f_{D^*}, we find that the resulting contribution from the radially excited states is very small at the level of this lower bound. For the pseudoscalar sum rule, one has to change the l.h.s. according to

$$f_D^2 \to f_D^2 + \left(\frac{m_{D'}}{m_D}\right)^4 f_{D'}^2 \exp\left(-\frac{m_{D'}^2 - m_D^2}{M^2}\right). \tag{4.4}$$

Numerically, that change is completely negligible: it adds less than 0.0016 GeV^2 (times the exponential factor, dependent on M^2, and which is smaller than 0.25) to $f_D^2 = 0.17^2 = 0.029$ GeV^2. Therefore, we have much room left even if the bound (4.2) were to be lowered. For the vector sum rule, the conclusion is quite similar. The substitution to be made is

$$f_{D^*}^2 \to f_{D^*}^2 + \left(\frac{m_{D^{*\prime}}}{m_D^*}\right)^2 f_{D^{*\prime}}^2 \exp\left(-\frac{m_{D^{*\prime}}^2 - m_{D^*}^2}{M^2}\right). \tag{4.5}$$

The change is numerically even smaller in this case.

In contrast to the LCSR for $f_D f_{D^*} g_{D^*D\pi}$, the radial excitation term does not improve the stability of the sum rules for the annihilation constants: indeed, the r.h.s. is decreasing with M^2, and the contribution from the radially excited state increases, since it is positive. But it does not deteriorate the stability, since it is very small.

5. Conclusion

The result of the proposed modification of the sum rule calculation is very encouraging. Introducing the radial excitation contribution significantly improves the value of $g_{D^*D\pi}$, and at the same time the stability of the sum rule with respect to the Borel parameter M^2. This means that the effect of such a radial state is not properly accounted by the standard perturbative continuum contribution. Unless one imposes an unreasonable degree of stability, one is not able to fix the magnitude of the radial contribution accurately. The latter may vary by more than a factor of two, which induces an important variation of $g_{D^*D\pi}$, around 30%. This should not be viewed as a problem since our main goal is to prove that a large value for $g_{D^*D\pi}$, i.e. in the range allowed by experiment, is possible in the LCSR approach. At the very least, we can say that the old solution with the low value of $g_{D^*D\pi}$ and no explicit radial excitation, is not compelling, and alternative ones with large values of $g_{D^*D\pi}$ are favored. Of course, one should wonder whether the success survives when the calculation of the theoretical side is improved. In particular, does it survive the introduction of the radiative corrections, which are large? Our answer is positive, but we would like to present it within a more extensive discussion of the parameters involved in the sum rule calculation, especially because the threshold parameter s_0 should depend on the fact that we separate an explicit contribution of some excited states.

The main reason for having such a large effect of radial excitations without exceedingly large couplings is that the *Borel exponential suppression effect is very weak in the LCSR*: at most a factor 0.5 (at the lower end of the allowed range) for the relative suppression of the radial excitation with respect to the ground state. The Borel suppression is much less effective than in the standard sum rules, e.g. for correlators leading to $f_{D^{(*)}}$. This is due to the additional factor of 1/2 in the exponent and to the fact that M^2 is twice larger on the mean in the allowed range (2 $\mathrm{GeV}^2 < M^2 < 4$ GeV^2, instead of 1 $\mathrm{GeV}^2 < M^2 <$ 2 GeV^2). At the lower end of the range, the suppression is around eight times less effective in the LCSR.

Of course, it would be good to examine whether one gets similar improvement for the other sum rule approaches to $g_{D^*D\pi}$. On the other hand, the situation encountered here underlines the usefulness of an alternative to the Borel transformation method, able to isolate with more efficiency various states. In some cases, the FESR proposed for the case of the quark mass sum rules by Kambor and Maltman [14] could be a good alternative.

Acknowledgments

We thank Alexander Khodjamirian for discussions and precious comments on the manuscript. We acknowledge the partial support from the EC contract HPRN-CT-2002-00311 (EURIDICE).

References

[1] V.M. Belyaev, V.M. Braun, A. Khodjamirian and R. Rückl, *$D^*D\pi$ and $B^*B\pi$ couplings in QCD*, *Phys. Rev.* **D 51** (1995) 6177 [hep-ph/9410280].

[2] A. Khodjamirian, R. Rückl, S. Weinzierl and O.I. Yakovlev, *Perturbative QCD correction to the light-cone sum rule for the $B^*B\pi$ and $D^*D\pi$ couplings*, *Phys. Lett.* **B 457** (1999) 245 [hep-ph/9903421].

[3] CLEO collaboration, S. Ahmed et al., *First measurement of* $\Gamma(D^{*+})$, *Phys. Rev. Lett.* **87** (2001) 251801 [hep-ex/0108013];
CLEO collaboration, A. Anastassov et al., *First measurement of* $\Gamma(D^{*+})$ *and precision measurement of* $M(D^{*+})$-$M(D^0)$, *Phys. Rev.* **D 65** (2002) 032003 [hep-ex/0108043].

[4] For a review, P. Colangelo and A. Khodjamirian, *QCD sum rules: a modern perspective*, hep-ph/0010175.

[5] D. Becirevic and A.L. Yaouanc, $\hat{g}$ *coupling* $(g_{B^*B\pi}, g_{D^*D\pi})$*: a quark model with Dirac equation*, *J. High Energy Phys.* **03** (1999) 021 [hep-ph/9901431].

[6] W. Jaus, *Consistent treatment of spin-1 mesons in the light-front quark model*, hep-ph/0212098.

[7] A. Abada, D. Becirevic, P. Boucaud, G. Herdoiza, J.P. Leroy, A. LeYaouanc, O. Pène, J. Rodriguez-Quintero, *First lattice QCD estimate of the* $g_{D^*D\pi}$ *coupling*, *Phys. Rev.* **D 66** (2002) 074504 [hep-ph/0206237].

[8] H.-C. Kim, *Re-analysis of the* $D^*D\pi$ *coupling in the light-cone QCD sum rules*, hep-ph/0206170.

[9] A. Khodjamirian, *QCD sum rules for heavy flavour physics*, *AIP Conf. Proc.* **602** (2001) 194 [hep-ph/0108205].

[10] C.A. Dominguez and M. Loewe, *Chiral and flavor* SU(2) *and* SU(3) *symmetry breaking in quantum chromodynamics*, *Phys. Rev.* **D 31** (1985) 2930;
M. Jamin and M. Munz, *The strange quark mass from QCD sum rules*, *Z. Physik* **C 66** (1995) 633 [hep-ph/9409335].

[11] D. Melikhov and O. Pène, *Is the resonance D(2637) really a radial excitation?*, *Phys. Lett.* **B 446** (1999) 336 [hep-ph/9809308].

[12] G. Burdman and J. Kambor, *Dispersive approach to semileptonic form-factors in heavy-to-light meson decays*, *Phys. Rev.* **D 55** (1997) 2817 [hep-ph/9602353].

[13] D. Becirevic and A.B. Kaidalov, *Comment on the heavy $\to$ light form factors*, *Phys. Lett.* **B 478** (2000) 417 [hep-ph/9904490].

[14] K. Maltman and J. Kambor, *Decay constants, light quark masses and quark mass bounds from light quark pseudoscalar sum rules*, *Phys. Rev.* **D 65** (2002) 074013 [hep-ph/0108227].

JHEP01(2003)009